Large time behavior of a solution to a two-scale problem as mathematical model for sulfate attack in sewer pipes

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In this talk we consider the following two-scale problem which is a mathematical model for concrete corrosion with sulfate attack in sewer pipes. Denote by Ω and Y_1 a macro domain $\Omega \subset \mathbb{R}^3$ and a micro domain $Y_1 \subset \mathbb{R}^3$, respectively, and $T > 0$. Let w_1, $w_2 : (0,T) \times \Omega \times Y_1 \to \mathbb{R}$, $w_3 : (0,T) \times \Omega \to \mathbb{R}$ and $w_4 : (0,T) \times \Gamma_1 \to \mathbb{R}$ be concentrations of H_2SO_4(aq), H_2S(aq), H_2S(g) and gypsum, respectively, where $\Gamma_1 \subset \partial\Omega$. Then these functions satisfy:

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\begin{aligned}
\partial_t w_1 - \nabla_y \cdot (d_1 \nabla_y w_1) &= -f_1(w_1) + f_2(w_2) \quad \text{in } (0,T) \times \Omega \times Y_1,\\
\partial_t w_2 - \nabla_y \cdot (d_2 \nabla_y w_2) &= f_1(w_1) - f_2(w_2) \quad \text{in } (0,T) \times \Omega \times Y_1,\\
\partial_t w_3 - \nabla \cdot (d_3 \nabla w_3) &= -\alpha \int_{\Gamma_2} (h_0 w_3 - w_2) \mathrm{d}\gamma_y \quad \text{in } (0,T) \times \Omega,\\
\partial_t w_4 &= \eta(w_1, w_4) \quad \text{on } (0,T) \times \Omega \times \Gamma_1,\\
w_j(0,x,y) &= w_{j0}(x,y),\ j = 1,2 \quad \text{in } \Omega \times Y_1,\\
w_3(0,x) &= w_{30}(x) \quad \text{in } \Omega,\ w_4(0,x,y) = w_{40}(x,y) \text{ on } \Omega \times \Gamma_1,\\
d_1 \nabla_y w_1 \cdot \nu(y) &= -\eta(w_1, w_4) \quad \text{on} (0,T) \times \Omega \times \Gamma_1,\\
d_1 \nabla_y w_1 \cdot \nu(y) &= 0 \quad \text{on } (0,T) \times \Omega \times \Gamma_2 \text{ and } (0,T) \times \Omega \times \Gamma_3,\\
d_2 \nabla_y w_2 \cdot \nu(y) &= 0 \quad \text{on } (0,T) \times \Omega \times \Gamma_1 \text{ and } (0,T) \times \Omega \times \Gamma_3,\\
d_2 \nabla_y w_2 \cdot \nu(y) &= \alpha(h_0 w_3 - w_2) \quad \text{on } (0,T) \times \Omega \times \Gamma_2,\\
d_3 \nabla w_3 \cdot \nu(x) &= 0 \quad \text{on } (0,T) \times \Gamma_N, w_3 = w_3^D \quad \text{on } (0,T) \times \Gamma_D,
\end{aligned}
$$

where Γ_1, Γ_2 and Γ_3 are subsets of ∂Y_1, disjoints and $\partial Y_1 = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3$, d_1 and d_2 are given positive functions on $\Omega \times Y_1$, f_1 and f_2 are continuous and increasing functions on $\mathbb{R}$, d_3 is a given positive function on Ω, α and h_0 are positive constants, η is a continuous function on $\mathbb{R}^2$, w_{j0} is a initial function for each $j = 1,2,3,4$, and w_3^D is a given function on $(0,T) \times \Gamma_D$, where $\Gamma_D \subset \partial\Omega$. This system was already studied in [1].

The purposes of this talk are to show the mathematical modeling process for the above system, and to establish existence and uniqueness of a solution and a large time behavior result. More precisely, $w_1(t) \to w_{1\infty}$, $w_2(t) \to w_{2\infty}$ and $w_3(t) \to w_{3\infty}$ in L^2, and $w_4(t) \to w_{4\infty}$ in L^1 as $t \to \infty$.

This is a joint work with Adrian Muntean in Eindhoven University of Technology.

References

[1] *V. Chalupecký, T. Fatima, A. Muntean*: Numerical study of a fast micro-macro mass transfer limit: The case of sulfate attack in sewer pipes. J. Math-for-Ind. *2B* (2010), 171–181.