On asymptotic behavior of solutions to Emden-Fowler type higher-order differential equations

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For the equation

$$y^{(n)} = p(x, y, y', \dots, y^{(n-1)})|y|^k \operatorname{sgn} y, \quad n \geq 2,\ k > 1, \tag{1}$$

asymptotic behavior of its positive solutions is investigated. For $n = 2$ (see [1]), $n = 3$ and $n = 4$ (see [2]) it was proved that under some conditions on the function $p(x, y, \dots)$ all such solutions behave as

$$y(x) = C(x^* - x)^{-\alpha}(1 + o(1)),\ x \to x^* - 0,\ \alpha = \frac{n}{k-1},\ C = \left(\frac{\alpha(\alpha+1)\dots(\alpha+n-1)}{\lim\limits_{\substack{x \to x^* \\ y_j \to \infty}} p(x, y_0, \dots, y_{n-1})} \right)^{\frac{1}{k-1}}. \tag{2}$$

Existence of solutions satisfying (2) was proved for arbitrary $n \geq 2$. For all $2 \leq n \leq 11$, $(n-1)$-parametric families of such solutions to equation (1) were proved to exist [2]. For equation (1) with $p(x, y, \dots) = 1$ it was proved [3] that for any N and $K > 1$ there exist an integer $n > N$ and $k \in \mathbf{R}$, $1 < k < K$, such that this equation has a positive solution with non-powered asymptotic behavior. Still it was not clear how large n should be for existence of that type of solutions.

Theorem. *Suppose* $12 \leq n \leq 14$. *Then there exists* $k > 1$ *such that equation* (1) *with* $p(x, y, \dots) \equiv 1$ *has a solution* $y(x)$ *satisfying*

$$y^{(j)}(x) = (x^* - x)^{-\alpha-j} h_j(\log(x^* - x)), \quad j = 0, 1, \dots, n-1,$$

with periodic positive non-constant functions h_j *on* $\mathbf{R}$. *(For* $n = 12$ *see* [4]*).*

Remark. Computer calculations give approximate values of k: if $n = 12$, then $k \approx 22.4$; if $n = 13$, then $k \approx 10.0$; if $n = 14$, then $k \approx 6.9$.

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References

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