A new error estimate for a fully finite element discretization scheme for parabolic equations using Crank-Nicolson method

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We consider conforming finite element method with piecewise polynomial space of degree k in space for solving the parabolic equations. The discretization in time is performed using the Crank-Nicolson method.

A new *a priori estimate* is proved. Thanks to this new *a priori estimate*, a new error estimate of order $h^{k+1} + \tau^2$, where h (resp. τ) is the mesh size of the spatial (resp. temporal) discretization, in the discrete norm of $\mathcal{W}^{1,\infty}(0,T;\, \mathcal{L}^2(\Omega))$ is proved. An $\mathcal{L}^\infty(0,T;\, \mathcal{H}^1(\Omega))$-error estimate of order $h^k + \tau^2$ is also shown.

References

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