On the existence of solutions of ordinary differential equations in Banach spaces

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In this talk we shall give sufficient conditions for the existence of solutions of a first order differential equation in a Banach space. We investigate the Cauchy problem

$$x' = f(t,x), \quad x(t_0) = x_0 \tag{1}$$

where E is a Banach space, B is the ball in E and $f : [0,a] \times B \mapsto E$ is a bounded continuous function. Our considerations are inspirated by the paper [1] concerning the unicity of solutions of the problem (1). We suppose that f satisfies generalized α-Nagumo condition $\lim_{r\to 0^+} \alpha(f((t-r,t+r) \cap [0,a] \times X)) \leq \frac{u'(t)}{u(t)}\omega(\alpha(X))$, where α is the measure of noncompactness, $X \subset B$ and $t \in (0,a)$, for some smooth function $u : [0,a] \to [0,\infty)$ with $u(0) = 0$ and $u'(t) > 0$ a.e. on $[0,a]$ and for some continuous and increasing function $\omega : [0,K] \to [0,\infty)$ which is null in 0 and positive every else, and also satisfies the integral inequality $\int_0^r \frac{\omega(s)}{s}\,\mathrm{d}s \leq r$, $r \in (0,K]$ $(K > 0)$. Moreover, assuming $\lim_{\substack{t\to 0^+ \\ r\to 0^+}} \frac{\alpha(f(t,B(0,r)))}{u'(t)} = 0$, where $B(0,r)$ is the ball with center 0 and radius r, we shall prove that there exists a compact subinterval J of $[0,a]$ such that the problem (1) has at least one solution defined on J. Our assumptions and proofs are expressed in terms of the measure of noncompactness.

References

[1] *A. Constantin*: On Nagumo's theorem. Proc. Japan Acad., Ser. A *86* (2010), 41–44.