

Multiplicity results for the scalar curvature equation

Matteo Franca

Department of Mathematics, Università Politecnica delle Marche (Ancona), Italy

franca@dipmat.univpm.it

In this talk we review some facts concerning positive solutions of linear and quasi-linear equation with a potential with mixed Sobolev growth, focusing in particular on radial solutions. Then we consider the so called scalar curvature equation

$$\Delta u(x) + K(|x|)u^{\sigma-1}(x),$$

where $\sigma = \frac{2n}{n-2}$, and we assume that $K(|x|) = k(|x|^\epsilon)$ is a singular perturbation of a positive constant. It is known that we have at least a ground state with fast decay for each positive critical point of k for ϵ small enough, see [1], [2]. In fact if we have just one critical point $k(r_0)$ and it is a maximum, we also have uniqueness, see [3]; surprisingly we show that if $k(r_0)$ is a minimum we have an arbitrarily large number of ground states with fast decay. The result has applications in differential geometry and quantum mechanics, and it is obtained using a dynamical system approach inspired by Melnikov theory. This is a joint work with prof. Isabel Flores from Universidad Tecnica Federigo Santa Maria in Valparaiso (Chile).

References

- [1] *M. Franca, R. Johnson*: Ground states and singular ground states for quasilinear partial differential equations with critical exponent in the perturbative case. *Adv. Nonlinear Stud.* 4 (2004) 93–120.
- [2] *R. Johnson, X. B. Pan, Y. F. Yi*: The Melnikov method and elliptic equations with critical exponent. *Indiana Univ. Math. J.* 43 (1994), 1045–1077.
- [3] *N. Kawano, E. Yanagida, S. Yotsutani*: Structure theorems for positive radial solutions to $\operatorname{div}(|Du|^{m-2}Du) + K(|x|)u^q = 0$ in R^n . *J. Math. Soc. Japan* 45 (1993) 719–742.