Abstract theory of variational inequality and application to nonlinear PDE

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The existence of solutions for a class of abstract evolution equations generated by maximal monotone operators with non-local convex constraint is considered. This result is a generalization of the theory of Lagrange multiplier related to variational inequalities [1], [3]. Based on the concept of optimization problems, the theory of Lagrange multiplier has strong relevance to various minimizing problems for some cost functionals with constraint. The objective of this paper is to give an extension of the well-known theory to a more general setting, in order to apply to nonlinear parabolic partial differential inclusions [2], [4].

Let $\mathcal{V}$ be a real reflexive and strictly convex Banach space and $\mathcal{V}^*$ be the dual space of $\mathcal{V}$, $\mathcal{A} : \mathcal{V} \to 2^{\mathcal{V}^*}$ be a maximal monotone operator, and $\Psi : \mathcal{V} \to [0, +\infty)$ be a continuous, convex and bounded functional with $D(\Psi) = \mathcal{V}$; hence $\Psi(B)$ is a bounded subset of $\mathbb{R}$ for each bounded subset $B \subset \mathcal{V}$. Then, consider the following problem to find an element $u \in \mathcal{V}$ and a number λ such that

$$\mathcal{A}u + \lambda \partial_* \Psi(u) \ni f \quad \text{in } \mathcal{V}^*,$$
$$\lambda \geq 0, \quad \Psi(u) - k_0 \leq 0, \quad \lambda(\Psi(u) - k_0) = 0,$$

where k_0 is given constant. An existence result with some comments are presented.

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References

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