The termination principle and bifurcation geometry of polynomial dynamical systems

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The global qualitative analysis of polynomial dynamical systems is carried out. To control all of their limit cycle bifurcations, especially, bifurcations of multiple limit cycles, it is necessary to know the properties and combine the effects of all of their rotation parameters. It can be done by means of the development of new bifurcational geometric methods based on the well-known Weierstrass preparation theorem and the Perko planar termination principle stating that the maximal one-parameter family of multiple limit cycles terminates either at a singular point which is typically of the same multiplicity (cyclicity) or on a separatrix cycle which is also typically of the same multiplicity (cyclicity) [1].

This principle is a consequence of the principle of natural termination which was stated for higher-dimensional dynamical systems by A. Wintner who studied one-parameter families of periodic orbits of the restricted three-body problem and used Puiseux series to show that in the analytic case any one-parameter family of periodic orbits can be uniquely continued through any bifurcation except a period-doubling bifurcation. The Wintner-Perko termination principle can be applied for studying multiple limit cycle bifurcations of planar polynomial dynamical systems [1].

If we do not know the cyclicity of the termination points, then, applying canonical systems with field rotation parameters, we use geometric properties of the spirals filling the interior and exterior domains of limit cycles. Applying this method, we have solved, e. g., Smale's Thirteenth Problem proving that the Liénard system with a polynomial of degree $2k+1$ can have at most k limit cycles [2]. Generalizing the obtained results, we have also solved the problem of the maximum number of limit cycles surrounding a singular point for an arbitrary polynomial system [2] and Hilbert's Sixteenth Problem for a general Liénard polynomial system with an arbitrary (but finite) number of singular points [3].

Finally, applying the same approach, we consider three-dimensional polynomial dynamical systems and, in particular, complete the strange attractor bifurcation scenario in the classical Lorenz system globally connecting the homoclinic, period-doubling, Andronov-Shilnikov, and period-halving bifurcations of its limit cycles.

References

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