Non-isolated T-periodic orbits of T-periodic differential equations through inverse Jacobi multipliers

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We consider a nonautonomous T-periodic differential system

$$\dot{x} = f(t,x), \tag{1}$$

where $f : \mathbb{R} \times \widetilde{U} \to \mathbb{R}^n$ is a C^1 function such that $f(\cdot, x)$ is T-periodic for each $x \in \widetilde{U}$ with $\widetilde{U} \subset \mathbb{R}^n$ open.

We denote the components $x = (x_1, \ldots, x_n)$ and $f = (f_1, \ldots, f_n)$. We associate to system (1) the vector field $\mathcal{X} = \partial_t + \sum_{i=1}^{n} f_i(t,x)\partial_{x_i}$ being $\operatorname{div} \mathcal{X} = \sum_{i=1}^{n} \partial f_i / \partial x_i$ its divergence. A C^1 function $V : \Omega \subset \mathbb{R} \times \widetilde{U} \to \mathbb{R}$ is said to be an inverse Jacobi multiplier for system (1) in the open set Ω if it is not locally null and it satisfies $\mathcal{X}V = V \operatorname{div} \mathcal{X}$. The function $1/V$ were introduced for the first time by Jacobi in [2]. We say that V is T-periodic in $\mathbb{R} \times U_0$ if $V(t,x) = V(t+T,x)$ for all $t \in \mathbb{R}$ and $x \in U_0 \subset \widetilde{U}$.

We assume that there is an open set $U \subset \widetilde{U}$ such that $[0,T] \subset I_{(0,x)}$ for any $x \in U$ where $I_{(0,x)}$ is the maximal interval of existence of the solution $\psi(t;0,x)$ of (1) with initial condition $\psi(0;0,x) = x$. Then we show how the existence of just one T-periodic solution $\psi(t;0,x)$ with $x \in U$ and sufficiently many independent T-periodic first integrals and nonvanishing T-periodic inverse Jacobi multipliers in $\mathbb{R} \times \widetilde{U}$ assures the existence of continua of T-periodic solutions of (1).

To prove the results we need first to find the relation

$$V(T, \Pi(x)) = V(0,x) \det D\Pi(x) \tag{2}$$

between V and the Poincaré translation map $\Pi : U \subset \widetilde{U} \to \widetilde{U}$ at time T defined by $\Pi(x) = \psi(T;0,x)$ that extends to arbitrary dimension the analogous fundamental relation for scalar equations proved in [1].

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References

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