On semi-discrete problems' analytic solutions for some equations of mathematical physics with periodic boundary conditions

Aigars Gedroics

Faculty of Physics and Mathematics, University of Latvia, Riga, Latvia

aigars.gedroics@lu.lv

This paper investigates finite difference schemes for problems of mathematical physics with periodic boundary conditions. Schemes with higher approximation order are created by finding circulant matrices which approximate the derivatives of the first and the second order. Formulas for finding eigenvalues of these matrices are created in a such form that the eigenvalues of higher approximation order can be calculated from the previous results. Formula for eigenvalues for $O(h^{2k})$ scheme approximating derrivative of the second order is in such form:

$$\mu_k = \frac{4}{h^2} \sum_{m=1}^{n} P_m \sin^2\left(\frac{\pi k}{N}\right)$$

$$P_m = \frac{2^{2m-1} \cdot ((m-1)!)^2}{(2m)!}$$

Multiple problems, i.e. heat transfer equations, hyperbolic type equations, Poisson's equations, are investigated, also with convection. The problems are modelled using method of lines. Discrete problems are created using spectral decomposition of discrete matrices or discrete Fourier series. Analytical solutions are found for the discrete model in most cases.

References

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