Centers in the trigonometric Abel equation

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We consider the trigonometric Abel equation

$$\frac{\mathrm{d}\rho}{\mathrm{d}\theta} = a_1(\theta)\rho^2 + a_2(\theta)\rho^3 \tag{1}$$

defined on the cylinder $(\rho, \theta) \in \mathbb{R} \times \mathbb{S}^1$ and where $a_1(\theta), a_2(\theta)$ are real trigonometric polynomials in θ with degree $\max\{\deg a_1, \deg a_2\} = d$.

By the standard uniqueness theorem, there is a unique solution of equation (1) with the prescribed initial value $\rho(0) = \rho_0$. We say that equation (1) determines a *center* if for any sufficiently small initial value $\rho(0)$ the solution of (1) satisfies $\rho(0) = \rho(2\pi)$. The *center problem* for equation (1) is to find conditions on the coefficients a_1, a_2 under which the equation has a center.

A particular class of centers, called *universal centers*, has been defined by A. Brudnyi in [2], see also the references therein and [3], [4]. This kind of centers were also known in the literature as composition centers, see [1]. We define

$$\tilde{a}_i(\theta) := \int_0^\theta a_i(s)\mathrm{d}s \quad \text{for } i = 1, 2.$$

We say that equation (1) has a *universal center* if there exists a trigonometric polynomial $q(\theta)$ and two polynomials $p_i(z) \in \mathbb{R}[z]$ such that $\tilde{a}_i(\theta) = p_i\left(q(\theta)\right)$, for $i = 1, 2$.

In [1], M. Blinov characterized all the centers of equation (1) when $a_i(\theta)$ are trigonometric polynomials of degree at most 2. He showed that all the centers of equation (1) with $d \leq 2$ are universal. In this communication we present the state of the art about the characterization of universal centers of equation (1) and we provide an example of a cubic Abel equation, that is $d = 3$, which determines a non-universal center.

References

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