Oscillation constant for half-linear equations with asymptotically almost periodic coefficients

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We study the half-linear difference equations

$$\Delta[r_k\Phi(\Delta x_k)] + \frac{\gamma s_k}{(k+1)^{(p)}}\Phi(x_{k+1}) = 0, \qquad \Phi(x) = |x|^{p-1}\operatorname{sgn} x, p > 1,$$

where γ is a real number, $\{r_k\}$ and $\{s_k\}$ are arbitrary asymptotically almost periodic sequences such that

$$\inf\{r_k\} > 0, \quad \limsup_{k\to\infty} s_k > 0,$$

and $k^{(\alpha)}$ is the so-called generalized power function (sometimes also called the falling factorial power). We prove that these equations are conditionally oscillatory; i.e., we prove the existence of a constant Γ (dependent on r_k and s_k) with the property that the given equation is oscillatory for $\gamma > \Gamma$ and non-oscillatory for $\gamma < \Gamma$. Further, we explicitly find this constant Γ. We also mention similar behaviour of the half-linear differential equations of the form

$$[r(t)\Phi(x')]' + \frac{\gamma s(t)}{t^p}\Phi(x) = 0.$$

This is a joint work with Michal Veselý.

References

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