Spiral-shaped solutions to crystalline motion with a moving tip

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Spiral patterns on a crystal surface are often observed when the crystal is growing up. These spirals are generated by a screw dislocation. We here treat a spiral as an infinite-length curve with one end-point in the plane. In this talk we call the end-point "tip" and consider that the curve describes a step-line of the crystal surface. In [1], W. K. Burton, N. Cabrera and F. C. Frank consider a motion of the step-line and proposed the motion equation: $V = U(1-\lambda\kappa)$. Here, V is the normal velocity of the step-line, U is a constant, λ is the critical radius and κ is the curvature of step-line. Here and hereafter we always assume $U > 0$ since we treat the growth case. According to this motion, a straight step-line moves forward with constant velocity U and a curved step-line becomes deformed by a curvature effect. Notice that by a forward motion of the step-line the crystal grows. However, the above equation does not include any anisotropic effect from a structure of the crystal and an anisotropy of the mobility. Thus, in this talk we consider the case that th interfacial energy density is crystalline and introduce an anisotropy of the mobility, that is, we consider a motion of spiral-shaped polygonal curves by a crystalline motion:

$$\beta(\boldsymbol{N}_j)V_j = U - H_j, \tag{1}$$

where, $V_j, \boldsymbol{N}_j$ and H_j denote a normal velocity, a unit normal vector and a crystalline curvature of the j-th facet of solution curve, respectively. To solve the problem, we need to give a motion of tip as a *boundary condition*. Unfortunately, the tip motion of real crystal has not been established yet. In this talk we give a movement of the tip, concretely, and we assume that the tip moves along a given closed curve for simplicity. Under the certain conditions on a tip motion, we show that a self-intersection of solution curve does not occur and also show that any facets never disappear during time evolution. And finally we obtain that the spiral solution exists globally in time.

References

[1] *W. K. Burton, N. Cabrera, F. C. Frank*: The growth of crystals and the equilibrium structure of their surfaces. Philos. Trans. Roy. Soc. London, Ser. A *243* (1951), 299–358.

[2] *H. Imai, N. Ishimura, T. K. Ushijima*: A crystalline motion of spiral-shaped curves with symmetry. J. Math. Anal. Appl. *240* (1999), 115–127.

[3] *H. Imai, N. Ishimura, T. K. Ushijima*: Motion of spirals by crystalline curvature. M2AN, Math. Model. Numer. Anal. *33* (1999), 797–806.