Positive solutions of the p-Laplace Emden-Fowler equation in hollow thin symmetric domains

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We study positive solutions of the p-Laplace Emden-Fowler equation,

$$-\Delta_p u = u^{q-1}, \quad u > 0 \quad (x \in \Omega), \qquad u = 0 \quad (x \in \partial\Omega),$$

where $\Delta_p u := \mathrm{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian, Ω is a bounded domain in $\mathbb{R}^N$ with $N \geq 2$. We assume that $2 \leq p < q < p^*$ with $p^* := Np/(N-p)$ if $p < N$ and $p^* := \infty$ if $N \leq p$. The Rayleigh quotient $R(u)$ and the Nehari manifold $\mathcal{N}$ are defined by

$$R(u) := \left(\int_\Omega |\nabla u|^p \,\mathrm{d}x \right) \left(\int_\Omega |u|^q \,\mathrm{d}x \right)^{-p/q},$$

$$\mathcal{N} := \{u \in W_0^{1,p}(\Omega) \setminus \{0\} \colon \int_\Omega (|\nabla u|^p - |u|^q) \,\mathrm{d}x = 0\}.$$

Let G be a closed subgroup of the orthogonal group $O(N)$. We call Ω a G invariant domain if $g(\Omega) = \Omega$ for all $g \in G$. We denote the set of G invariant functions in $\mathcal{N}$ by $\mathcal{N}(G)$. We call u a G invariant least energy solution if it minimizes the Rayleigh quotient R in $\mathcal{N}(G)$. Let H and G be two closed subgroups of the orthogonal group $O(N)$ such that $H \subsetneq G$. We prove that an H invariant least energy solution is not G invariant if $H(x) \subsetneq G(x)$ for $x \in \overline{\Omega}$ and Ω is a hollow thin G invariant domain. Here $H(x)$ denotes the orbit of H through x., i.e., $H(x) := \{hx \colon h \in H\}$.