Periodic differential operators with asymptotically predefined spectral gaps

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It is well-known that the spectrum of a self-adjoint periodic differential operator is a union of compact intervals called *bands*. The neighbouring bands may overlap, otherwise we have a *gap* in the spectrum, i.e. an open interval that does not belong to the spectrum but its ends belong to it. In general the presence of gaps in the spectrum is not guaranteed.

Our goal is to construct periodic differential operators of various types with spectral gaps that are asymptotically close to predefined intervals. Namely, in the talk we discuss the following theorem absorbing the results of [1]–[3].

Theorem. *We denote by L^n_{per} $(n \in \mathbb{N})$ one of the following sets of operators:*

- *Laplace-Beltrami operators on n-dimensional $\mathbb{Z}^n$-periodic Riemannian manifolds,*
- *Elliptic operators in $\mathbb{R}^n$ in divergent form with $\mathbb{Z}^n$-periodic and piecewise constant coefficients,*
- *Neumann Laplacians in $\mathbb{Z}^n$-periodic domains in $\mathbb{R}^n$.*

Let (α_j, β_j) $(j = 1, \ldots, m,\ m \in \mathbb{N})$ be arbitrary finite intervals belonging to $(-\infty, 0]$. Let $\mathcal{I} \subset (-\infty, 0]$ be an arbitrarily large compact interval containing the union of these intervals. Let $n \in \mathbb{N} \setminus \{1\}$.

Then one can construct a family $\left\{A^\varepsilon \in L^n_{\mathrm{per}}\right\}_\varepsilon$ such that the spectrum of the operator A^ε has the following structure in $\mathcal{I}$:

$$\sigma(A^\varepsilon) \cap \mathcal{I} = \mathcal{I} \setminus \bigcup_{j=1}^{m} (\alpha_j^\varepsilon, \beta_j^\varepsilon)$$

where the intervals $(\alpha_j^\varepsilon, \beta_j^\varepsilon)$ satisfy

$$\forall j = 1, \ldots, m: \ \lim_{\varepsilon \to 0} \alpha_j^\varepsilon = \alpha_j, \ \lim_{\varepsilon \to 0} \beta_j^\varepsilon = \beta_j.$$

The idea how to construct the family $\{A^\varepsilon\}_\varepsilon$ is based on a concept of trap-like media (see §5.4 in [4]).

References

[1] *A. Khrabustovskyi*: Periodic Riemannian manifold with preassigned gaps in spectrum of Laplace-Beltrami operator. J. Differ. Equations *252* (2012), 2339–2369.

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[4] *V. Marchenko, E. Khruslov*: Homogenization of Partial Differential Equations. Birkhauser, Boston, 2006.