Connecting orbits for nonlinear evolution equations at resonance

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Assume that $A : X \supset D(A) \to X$ is a positive definite sectorial operator on a Banach space X with compact resolvents and let X^α, where $\alpha \in [0, 1)$, be a fractional power space given by $X^\alpha := D(A^\alpha)$. We shall consider differential equations of the form

$$\dot{u}(t) = -Au(t) + \lambda u(t) + F(u(t)), \quad t > 0 \tag{1}$$

$$\ddot{u}(t) = -Au(t) - cA\dot{u}(t) + \lambda u(t) + F(u(t)), \quad t > 0 \tag{2}$$

where $c > 0$, λ is a real number and $F : X^\alpha \to X$ is a continuous map. They are abstract formulation of many parabolic pde's including the nonlinear heat equation and strongly damped wave equation

$$u_t(x,t) = \Delta u(x,t) + \lambda u(x,t) + f(x, u(x,t)), \quad t \geq 0,\ x \in \Omega \tag{3}$$

$$u_{tt}(x,t) = \Delta u(x,t) + c\Delta u_t(x,t) + \lambda u(x,t) + f(x, u(x,t)), \quad t \geq 0,\ x \in \Omega \tag{4}$$

where Ω is an open bounded subset of $\mathbb{R}^n$ $(n \geq 1)$, Δ is a Laplace operator with the Dirichlet boundary conditions and $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is a continuous map. Our goal is study the existence of connecting orbits for the above equations in the case of *the resonance at infinity*, that is, $\ker(\lambda I - A) \neq \{0\}$ and F is bounded. The main difficulty lies in the fact that, due to the presence of resonance, the equations (1) and (2) may not have connecting orbits for general perturbation F. To overcome this difficulty we provide geometrical assumptions on nonlinearity F and use them to prove theorems determining the Conley index for equations (1) and (2) on sufficiently large ball. The methods that we will use involve the application of homotopy invariants such as Rybakowski version of Conley index (see [4]) to semiflows associated with these equations. Furthermore, we prove that if F is a Niemycki operator associated with f:

$$F(u)(x) := f(x, u(x)) \quad \text{for } x \in \Omega,$$

then introduced geometrical assumptions generalize well known *Landesman-Lazer* and *strong resonance* conditions (see [1], [2]). Finally, we use the obtained results and provide applications to study the existence of connecting orbits for equations (3) and (4).

References

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