Quasilinear elliptic equations with positive exponent on the gradient

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We study the existence and nonexistence of positive, spherically symmetric solutions of a quasilinear elliptic equations

$$
\begin{cases} -\Delta_p u = \tilde{g}_0 |x|^m + \tilde{f}_0 |\nabla u|^{e_0} \quad \text{in } B \setminus \{0\}, \\ \quad u = 0 \text{ on } \partial B, \\ u(x) \text{ spherically symmetric and decreasing,} \end{cases} \tag{1}
$$

involving p-Laplace operator, with an arbitrary positive growth rate e_0 on the gradient on the right-hand side. We show that $e_0 = p - 1$ is the critical exponent: for $e_0 < p - 1$ there exists a strong solution for any choice of the coefficients which is a known result, while for $e_0 > p-1$ we have existence-nonexistence splitting of the coefficients $\tilde{f}_0$ and $\tilde{g}_0$. The elliptic problem is studied by relating it to the corresponding singular ODE of the first order. We give sufficient conditions for a strong radial solution to be a weak solution. We also examined when ω-solutions of (1), defined in the paper, are weak solutions. We found conditions under which the strong solutions are the weak solutions in the critical case of $e_0 = p-1$. This is a joint work with Darko Žubrinić.

References

[1] *H. Brezis*: Analyse Fonctionelle. Théorie et Applications. Masson, Paris, 1983.

[2] *D. Gilbarg, N. S. Trudinger*: Elliptic Partial Differential Equations of Second Order. Springer-Verlag, Berlin, 1983.

[3] *L. Korkut, M. Pašić, D. Žubrinić*: A singular ODE related to quasilinear elliptic equations. Electron. J. Differ. Equ. *2000* (2000), 37 pp.

[4] *D. Žubrinić*: Positive solutions of quasilinear elliptic systems with the natural growth in the gradient. Rend. Ist. Mat. Univ. Trieste *32* (2000), 65–102 (2001).