The generalized Krasnosel'skii formula for semilinear differential equations and periodic solutions

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In the talk we will study the existence and the behavior of nontrivial solutions to the following parameterized semilinear differential equation

$$\dot{u} \in Au + F_\lambda(t,u),\ t \in J,\ u \in E,$$

where $J := [0, +\infty)$, $\lambda \in \Lambda$, Λ is a compact subset of R^k, $A : D(A) \to E$, $D(A) \subset E$, E is a Banach space, is the infinitesimal generator of a C_0-semigroup $S = \{S(t)\}_{t\geq 0}$ of bounded linear operators on E, $F : \Lambda \times J \times E \to E$ is a continuous map or a set-valued weakly upper semicontinuous map with convex weakly compact values. It is to be noted that many (parameterized) partial differential equations (or systems of such equations), e.g. of parabolic (in particular nonlinear reaction-diffusion equations) or hyperbolic type (or inclusions), can be transformed so as to have this form. With this problem one associates the Poincaré translation operator Φ_t, where $t > 0$, which assigns to each initial value $x \in E$ the set of values $u(t)$, where u the (mild) solution starting at x. We show that, in the spirit of the Krasnosel'skii formula which relates the Brouwer degree of the right-hand side of an ordinary differential equation in R^n with the Brouwer fixed point index of the Poincaré operator, also in the considered situation the right-hand side of the equation is in an appropriate sense homotopic to Φ_t with sufficiently small $t > 0$, which translates to the formula relating the Leray-Schauder fixed point index of Φ_t with the appropriately defined topological degree of the right hand side. The key property of Φ_t, $t > 0$, is its compactness. As it appears, under some additional assumptions, e.g. the compactness of the semigroup generated by A or the appropriate relation between the growth bound ω of the semigroup S and the contractivity constant (with respect to the Hausdorff measure of noncompactness) of F, the Poincaré operator Φ_t inherits the sufficient compactness properties allowing to introduce its homotopy invariants detecting fixed points (and, therefore, periodic solutions if $t = T$). In order to get criteria for the existence of periodic solutions and their bifurcations, stated in terms of the topological behavior of the right hand side of the equation and its linearization, one uses the Alexander invariant, responsible for bifurcation phenomena, or the so-called averaging principles (generalizing some results of Henry, Bogolubow) allowing to determine the local fixed point index of Φ_T and its bifurcation index.

The situation subject to some state constraints will also be presented.

References

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