

Regularity of solutions of 3D Navier-Stokes equations in a Lipschitz domain for small data

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We consider the global existence of strong solutions of the 3D incompressible Navier-Stokes equations in a bounded Lipschitz domain under Dirichlet boundary condition. We present by a very simple argument that a strong solution exists globally when the product of L^2 norms of the initial velocity and the gradient of the initial velocity and $L^{p,2}$, $p \geq 4$ norm of the forcing function are small enough. Our condition is scale invariant and implies many typical known global existence results for small initial data including the sharp dependence of the bound on the volume of the domain and viscosity. We also present a similar result in the whole domain with slightly stronger condition for the forcing.

References

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