

A new solution operator for p -Laplacian systems with sign-changing singular weights

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We construct a new solution operator for problem

$$(A) \quad \begin{cases} -\psi_p(u)' = g, & t \in (0, 1), \\ u(0) = 0 = u(1), \end{cases}$$

where $\psi_p : \mathbb{R}^N \rightarrow \mathbb{R}^N$ is defined by $\psi_p(x) = |x|^{p-2}x$. $g : (0, 1) \rightarrow \mathbb{R}^N$ may be singular at 0 or/and 1 and changes a sign. Moreover g may not be in $L^1(0, 1)$. This operator extends the ones introduced by Manásevich-Mawhin [1] in the $g \in L^1(0, 1)$ case, and by Agarwal-Lü-O'Regan [2] in the case of positive solutions.

We also show the complete continuity of the solution operator for the problem

$$(P_\lambda) \quad \begin{cases} \psi_p(u)' + \lambda h(t) \cdot f(u(t)) = 0, & t \in (0, 1), \\ u(0) = 0 = u(1), \end{cases}$$

where $\lambda > 0$ is a parameter, $h : (0, 1) \rightarrow \mathbb{R}^N$ may be beyond of $L^1(0, 1)$, $f \in C(\mathbb{R}^N, \mathbb{R}^N)$, and we denote $x \cdot y := (x_1y_1, x_2y_2, \dots, x_Ny_N)$. As an application, we show the existence of non-trivial solutions of problem (P_λ) , under additional growth conditions on f .

References

- [1] R. Manásevich, J. Mawhin: Periodic solutions of nonlinear systems with p -Laplacian-like operators. JDE 145 (1998), 367–393.
- [2] R. P. Agarwal, H. Lü, D. O'Regan: Eigenvalues and the one-dimensional p -Laplacian. J. Math. Anal. Appl. 266 (2002), 383–400.