Multiple positive solutions for a higher-order multi-point boundary value problem

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This is a joint work with Prof. Johnny Henderson (Baylor University, Waco, Texas, USA).
We consider the system of nonlinear higher-order ordinary differential equations

$$\text{(S)} \qquad \begin{cases} u^{(n)}(t) + f(t, v(t)) = 0, \;\; t \in (0,T), \\ v^{(m)}(t) + g(t, u(t)) = 0, \;\; t \in (0,T), \end{cases}$$

with the multi-point boundary conditions

$$\text{(BC)} \qquad \begin{cases} u(0) = \sum_{i=1}^{p} a_i u(\xi_i), \;\; u'(0) = \cdots = u^{(n-2)}(0) = 0, \;\; u(T) = \sum_{i=1}^{q} b_i u(\eta_i), \\ v(0) = \sum_{i=1}^{r} c_i v(\zeta_i), \;\; v'(0) = \cdots = v^{(m-2)}(0) = 0, \;\; v(T) = \sum_{i=1}^{l} d_i v(\rho_i), \end{cases}$$

where $n, m \in N$, $n, m \geq 2$, $p, q, r, l \in N$. Under sufficient conditions on f and g, we prove the existence and multiplicity of positive solutions of the problem (S)–(BC), by applying the fixed point index theory (see [1]). For $n = m = 2$, the system (S) with the boundary conditions $\alpha u(0) - \beta u'(0) = \sum_{i=1}^{p} a_i u(\xi_i)$, $\gamma u(T) + \delta u'(T) = \sum_{i=1}^{q} b_i u(\eta_i)$, $\tilde{\alpha} v(0) - \tilde{\beta} v'(0) = \sum_{i=1}^{r} c_i v(\zeta_i)$, $\tilde{\gamma} v(T) + \tilde{\delta} v'(T) = \sum_{i=1}^{l} d_i v(\rho_i)$, is also investigated.

References

[1] *J. Henderson, R. Luca*: Existence and multiplicity of positive solutions for a system of higher-order multi-point boundary value problems. Advances in Dynamical Systems and Applications, to appear.