Approximations of quantum-graph vertex couplings by singularly scaled potentials

Stepan S. Manko
Czech Technical University in Prague, Czechia
stepan.manko@gmail.com

The talk is based on a joint project with P. Exner [3]. We examine Schrödinger operators on n-edges star metric graphs with potentials of the form $\lambda(\varepsilon)\varepsilon^{-2}Q(\varepsilon^{-1}x)$ and indicate the norm-resolvent limits of these Hamiltonians. In dimension 1, it is well known that such operators have a nontrivial limit only if the potential Q generates a zero-energy resonance, i.e., a bounded solution to the zero-energy Schrödinger equation with the potential Q. In the case of graph, the zero-energy Schrödinger equation may have $m \in \{1, \ldots, n-1\}$ bounded solutions, and we will generate different couplings at the graph vertices depending on the number m of zero-energy resonances [2], [3]. We next discuss the stationary scattering on the potential $\lambda(\varepsilon)\varepsilon^{-2}Q(\varepsilon^{-1}x)$ and show that the corresponding scattering matrices converge as $\varepsilon \to 0$ to those for the limiting Hamiltonians [1], [3]. Thus such a potential is generically nontransparent in the limit $\varepsilon \to 0$, while it admits partial transmission when the zero-energy resonances exist. Finally, we will describe low- and high-energy behavior of the limiting scattering amplitudes.

References

[1] *S. Manko*: On δ'-like potential scattering on star graphs. J. Phys. A, Math. Theor. *43* (2010), Article ID 445304.

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[3] *P. Exner, S. Manko*: Approximations of quantum-graph vertex couplings by singularly scaled potentials. In preparation.