Sharp estimate of the spreading speed determined by nonlinear free boundary problems

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In this talk we are interested in obtaining sharp estimates for the spreading speed determined by the following free boundary problem:

$$\text{(FBP)} \qquad \begin{cases} u_t - u_{xx} = f(u), & t > 0,\ g(t) < x < h(t), \\ u(t, g(t)) = u(t, h(t)) = 0, & t > 0, \\ g'(t) = -\mu u_x(t, g(t)), & t > 0, \\ h'(t) = -\mu u_x(t, h(t)), & t > 0, \\ -g(0) = h(0) = h_0, u(0, x) = u_0(x), & -h_0 \le x \le h_0, \end{cases}$$

where $x = h(t)$ and $x = g(t)$ are the moving boundaries to be determined together with $u(t,x)$, μ is a given positive constant, $f : [0,\infty) \to \mathbb{R}$ is C^1, $f(0) = 0$ and is of monostable, or bistable, or of combustion type. The initial function u_0 belongs to $\mathscr{X}(h_0)$ for some $h_0 > 0$, where

$$\mathscr{X}(h_0) := \{\phi \in C^2[-h_0, h_0]\colon \phi(-h_0) = \phi(h_0) = 0,\ \phi'(-h_0) > 0,\ \phi'(h_0) < 0,\ \phi(x) > 0 \quad \text{in } (-h_0, h_0)\}.$$

For any $h_0 > 0$ and $u_0 \in \mathscr{X}(h_0)$, a triple $(u(t,x), g(t), h(t))$ is a (classical) solution to (FBP) for $0 < t \le T$ if it belongs to $C^{1,2}(G_T) \times C^1[0,T] \times C^1[0,T]$ and all the identities in (FBP) are satisfied pointwisely, where

$$G_T := \{(t,x)\colon t \in (0,T],\ x \in [g(t), h(t)]\}.$$

This problems may be used to describe the spreading of a biological or chemical species, with the free boundaries representing the expanding fronts. For monostable, bistable and combustion types of nonlinearities, Du and Lou [1] obtained rather complete description of the long-time dynamical behavior of the problem and revealed sharp transition phenomena between spreading ($\lim_{t\to\infty} u(t,x) = 1$) and vanishing ($\lim_{t\to\infty} u(t,x) = 0$). They also determined the asymptotic spreading speed of the fronts by making use of semi-waves when spreading happens. In this talk, we give a much sharper estimate for the spreading speed of the fronts than that in [1], and describe how the solution approaches the semi-wave when spreading happens. This is a joint reserch with Y. Du and M. Zhou based on [2].

References

[1] *Y. Du, B. Lou*: Spreading and vanishing in nonlinear diffusion problems with free boundaries. Preprint. (arXiv:1301.3373v1)

[2] *Y. Du, H. Matsuzawa, M. Zhou*: Sharp estimate of the spreading speed determined by nonlinear free boundary problems. Submitted.