

Properties of monodromic singularities on center manifolds in \mathbb{R}^3 characterized from Lie symmetries

Susanna Maza

Departament de Matemàtica, Universitat de Lleida, Spain
smaza@matematica.udl.cat

In this work we study the nondegenerate center problem on a center manifold of an analytic, autonomous differential system in \mathbb{R}^3 . Thus, in some linear coordinates and rescaling the time if necessary, we consider systems in the form

$$(1) \quad \dot{x} = -y + F_1(x, y, z), \quad \dot{y} = x + F_2(x, y, z), \quad \dot{z} = \lambda z + F_3(x, y, z),$$

where $\lambda \in \mathbb{R} \setminus \{0\}$, $F = (F_1, F_2, F_3) : U \rightarrow \mathbb{R}^3$ is real analytic on the neighborhood of the origin $U \subset \mathbb{R}^3$ with $F(0) = 0$ and whose Jacobian matrix $DF(0) = 0$.

Our approach is different from the classical one in the sense that we do not compute the Poincaré-Liapunov constants of the reduced system to the 2-dimensional center manifold $W^c(0)$ at the origin. Instead, we study the *center problem* (to decide if all the orbits on $W^c(0)$ near the origin are periodic or if they spiral around it) using infinitesimal generators \mathcal{Z} of *Lie symmetries* of the associated vector field \mathcal{X}_λ to system (1). In particular, we shall use the sets $C(\mathcal{X}_\lambda, U)$ and $N(\mathcal{X}_\lambda, U)$ of analytic *centralizers* (commuting vector fields) and *normalizers* of \mathcal{X}_λ on U , respectively.

We characterize when the origin of (1) is either a center on the unique center manifold or a saddle-focus (that is a focus on each center manifold) through the structure of the elements of $C(\mathcal{X}_\lambda, U)$ and $N(\mathcal{X}_\lambda, U)$. The results obtained are also useful to detect additional properties of the centers such as its analytical linearization or orbital linearization. This is a joint work with Isaac A. García from Universitat de Lleida and Douglas S. Shafer from University of North Carolina at Charlotte, see [1] for details.

References

- [1] I. A. García, S. Maza, D. S. Shafer: Properties of monodromic points on center manifolds in \mathbb{R}^3 via Lie symmetries. To appear.