Functional-differential equations with Riemann-Liouville fractional integrals in the nonlinearities

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We discuss the following initial value problem

$$\dot{x}(t) = Ax(t) + f(t, x(t), x_t, (I^{\alpha_1}x)(t), \ldots, (I^{\alpha_m}x)(t)),\ t > 0, \tag{1}$$

$$x(t) = \Phi(t),\ t \in [-r, 0], \tag{2}$$

where $r > 0$, $\Phi \in C := C([-r, 0], X)$, $x(t) \in X, X$ is a Banach space, $x_t \in C$, $x_t(\Theta) = x(t + \Theta)$, $t > 0$, $\Theta \in [-r, 0], A$ is a linear map,

$$(I^{\alpha_i}x)(t) := \frac{1}{\Gamma(\alpha_i)} \int_0^t (t-s)^{\alpha_i - 1} x(s) \mathrm{d}s,\ 0 < \alpha_i < 1,\ i = 1, 2, \ldots, m \tag{3}$$

are the Riemann-Liouville fractional integrals of the function $x(t)$.

We give some conditions on A and on the nonlinearity f, sufficient for the nonexistence of blowing-up mild solutions of the initial value problem (1), (2). These results are proved by an application of a nonlinear integral inequality with weakly singular kernel (see [4], [5], [6]).

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