Spectral comparison in a reaction-diffusion system

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We are concerned with the two-component system of reaction-diffusion equations,

$$\begin{cases} u_t = d\Delta u - g(u+\gamma v) + v, \\ v_t = \Delta v + g(u+\gamma v) - v, \end{cases}$$

in a bounded domain Ω with the homogeneous Neumann boundary conditions, where $0 < d < 1$ and g is of class C^1 bounded by a linear growth function. This system has conservation of a mass, namely, the integral of $u + v$ over the domain is preserved in t. We first show that the system possesses a Lyapunov function for any $\gamma \in [0, 1]$. By this result the omega-limit set of any bounded orbit in a phase space consists of equilibrium solutions. Next we assume $\gamma = 1$ with a fixed spatial average of $u + v$ and show that the stationary problem is reduced to the following scalar elliptic equation for $w = u + v$ with a nonlocal term:

$$\Delta w - \tilde{g}(w) + \frac{1}{|\Omega|}\int_\Omega \tilde{g}(w)\,\mathrm{d}x = 0, \qquad \tilde{g}(w) := \frac{1-d}{d}g(w) + w$$

subject to the Neumann boundary condition. This scalar equation is the Euler-Lagrange equation of an energy functional in $H^1(\Omega)$ with a constraint. Our main result tells that the dimension of the unstable manifold of any equilibrium solution to the system coincides with the Morse index of the corresponding critical point of the energy. This result is proved by the spectral comparison argument for the linearized eigenvalue problem of the equilibrium solution. This talk is based on the joint work with Shuichi Jimbo (Hokkaido University).

References

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