Solvability of mathematical modeling for brewing process of Japanese Sake with unknown finish time

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Our main objective of this talk is discussing the following mathematical model for brewing process of Japanese Sake.

$$
\begin{cases}
\frac{\partial u_1}{\partial t} = k_1 \triangle_N u_1 + c_1(\theta) u_1 - c_3 u_2 & \text{in } \mathrm{Q} := (0, \mathrm{T}) \times \Omega \\
\frac{\partial u_2}{\partial t} = k_2 \triangle_N u_2 - c_4 u_1 + c_2(\theta) u_2 & \text{in } \mathrm{Q} \\
(u_1, u_2) \in K_0(\theta) & \text{in } \mathrm{Q} \\
\frac{\partial \theta}{\partial t} = k_3 \triangle \theta + h(\mathcal{L} u) & \text{in } \mathrm{Q} \\
\text{Dirichlet or Robin boundary conditions are assigned for } \theta & \text{on } \Gamma := (0, \mathrm{T}) \times \partial\Omega \\
\frac{\partial a_1}{\partial t} = k_4 \triangle_N a_1 - c_9 a_1 u_1 & \text{in } \mathrm{Q} \\
\frac{\partial a_4}{\partial t} = k_7 \triangle_N a_4 + c_{12} a_1 u_1 - (c_{13} u_2 + c_{14} u_1) a_4 + f_2 & \text{in } \mathrm{Q}
\end{cases}
$$

where Ω is a bounded domain with smooth boundary in $\mathbf{R}^N$ $(N = 1, 2, 3)$. We prescribe homogeneous Neumann boundary condition for equations that contain the $\triangle_N$ term.

Brewing process of Sake has 5 stages of fermenting in general, and this system represents the first step of brewing process. We interested in the solvability and behavior of solutions of this model from mathematical point of view, and reproducibility of phenomena from physical point of view.

In brewing process, we make stopped the fermenting when the product reaches expected conditions. Saying with mathematical expression, the finish time T depends upon the solution, so we have to examine our system in such situations.

In this talk, we show the solvability with non-homogeneous Dirichlet boundary condition for θ, unique existence of the solution with Robin boundary condition, and some properties of solutions in certain situations.

This is a joint work with Prof. Akio Ito (Kinki University, Japan) and Prof. Nobuyuki Kenmochi (Bukkyo University, Japan).

References

[1] *A. Ito, N. Kenmochi, Y. Murase*: Mathematical model for brewing process of Japanese Sake and its analysis. Preprint.