On a linear fractional difference equation

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We shall discuss the solvability of an initial value problem for linear fractional difference equation in the form

$$\sum_{j=1}^{\lceil\alpha\rceil} p_{\lceil\alpha\rceil-j+1}(t)\, {}_0\nabla_h^{\alpha-j+1} y(t) + p_0(t)y(t) = 0, \quad t \in h\mathbb{Z}\,,\ t \geq (\lceil\alpha\rceil+1)h,$$

$${}_0\nabla_h^{\alpha-j} y(t)\big|_{t=\lceil\alpha\rceil h} = y_{\alpha-j}, \quad j = 1, 2, \ldots, \lceil\alpha\rceil,$$

where $\alpha \in \mathbb{R}^+$, $\lceil\alpha\rceil \in \mathbb{Z}^+$ is the ceiling of α, ${}_0\nabla_h^{\gamma}$ ($\gamma \in \mathbb{R}^+$) is the fractional backward h-difference, $p_i(t)$ ($i = 0, \ldots, \lceil\alpha\rceil$) are coefficients satisfying suitable conditions and $y_{\alpha-j}$ are arbitrary real constants. The existence and uniqueness as well as the structure of solution are presented. Restricting to a simplified (two-term) equation, we show an explicit form of the solution via discrete Mittag-Leffler functions. This is a joint research with J. Čermák and T. Kisela.

References

[1] *J. Čermák, T. Kisela, L. Nechvátal*: Discrete Mittag-Leffler functions in linear fractional difference equations. Abstr. Appl. Anal. *2011* (2011), 21 pages.

[2] *J. Čermák, L. Nechvátal*: On (q, h)-analogue of fractional calculus. J. Nonlinear Math. Phys. *17* (2010), 51–68.