Blow-up phenomena for Dullin-Gottwald-Holm equation with dissipative term

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We consider the following Cauchy problem

$$u_t - u_{xxt} + k(u - u_{xx})_x + 3uu_x + \lambda\,(u - u_{xx}) = 2u_x u_{xx} + uu_{xxx}, \tag{1}$$

$$u\,(0, x) = u_0(x), \tag{2}$$

where k, $\lambda > 0$.

The equation under consideration is a special case of the Dullin-Gottwald-Holm equation

$$u_t - \alpha^2 u_{xxt} + ku_x + 3uu_x + \gamma u_{xxx} = \alpha^2(2u_x u_{xx} + uu_{xxx}),$$

with weakly dissipative term $\lambda(u - u_{xx})$. Dullin-Gottwald-Holm equation describing the unidirectional propagation of surface waves in a shallow water regime was derived by the method of asymptotic analysis and a near-identity normal form transformation from water wave theory, combining the linear dispersive of the KdV equation with the nonlinear dispersion of the Camassa-Holm equation. It is completely integrable and its travelling wave solutions contains both the KdV solitons and the CH peakons as limiting cases.

We investigate the blow-up phenomena of the problem (1)–(2). Note that the obtained blow-up criterion and the used technique are different from previous ones.