The size of vorticity and its connection with pressure in nonlinear Navier-Stokes equations

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By studying the pressure equation in a nonlinear Navier-Stokes equation, it is proved that vorticity satisfies the relation $\int_\Omega |\omega|^2 \, \mathrm{d}x = \sum_{i=1}^{3} \int_\Omega |\nabla u_i|^2 \, \mathrm{d}x$, where $\Omega \subset \mathbb{R}^3$ can be bounded or the whole space, $\omega = \nabla \times u$ is the vorticity, and (u, p) corresponds to a solution of the nonlinear steady or unsteady Navier-Stokes equation; in the unsteady case the relation is satisfied at each time.

Previous relation comes from the elliptic problem that pressure satisfies. Eventually, with a deeper analysis of this Poisson problem, improvements in the regularity of pressure can be achieved, and so partial regularity results on the Navier-Stokes equation.

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