

On the stability of the initial conditions for the parabolic Gelfand Problem

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The parabolic Gelfand Problem corresponds to a parabolic quasi-linear reaction-diffusion equation, where the reaction term is the exponential function. This problem has important applications in *thermal runaway* concerning mathematical modelling of combustion processes, see [1]. The blow-up problem for this problem has been deeply studied concerning blow-up time, blow-up set and blow-up profiles, but surprisingly an analysis on which initial conditions develop or not blow-up is not available in the literature. Actually, there are no available criteria on the initial conditions that ensures development of blow-up.

In this work are presented very define criteria on initial conditions which determine development of blow-up. The first criterion is very general, while the second and third are given in terms of steady solutions. The three criteria describe, partially, the unstable manifold of the nonlinear semigroup behind the PDE. Numerical trials supporting the results are also presented.

References

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