Attractivity implies stability for half-linear differential systems with time-varying coefficients

Masakazu Onitsuka
Department of Applied Mathematics, Okayama University of Science, Japan
onitsuka@xmath.ous.ac.jp

We consider the nonautonomous differential system

$$
\begin{aligned}
x' &= -e(t)x + f(t)\phi_{p^*}(y),\\
y' &= -g(t)\phi_p(x) - h(t)y,
\end{aligned}
\tag{1}
$$

where p and p^* are positive numbers satisfying $1/p + 1/p^* = 1$, and $\phi_q(z) = |z|^{q-2}z$ for $q = p$ or $q = p^*$. This system is referred to as a half-linear system. In the special case in which $p = 2$, system (1) becomes the linear system

$$
\mathbf{x}' = A(t)\mathbf{x}, \qquad A(t) = \begin{pmatrix} -e(t) & f(t) \\ -g(t) & -h(t) \end{pmatrix},
\tag{2}
$$

where $\mathbf{x} = {}^t(x, y)$. Let $X(t)$ be a fundamental matrix for (2). As is well known, the zero solution of (2) is attractive if and only if $\|X(t)\| \to 0$ as $t \to \infty$, and the zero solution of (2) is stable if and only if $\|X(t)\|$ is bounded. Clealy, if the zero solution of (2) is attractive, then it is stable.

In the general case, where $p \neq 2$, however, the concept of fundamental matrices does not apply, because the sum of two solutions of (1) is not always a solution of (1), namely, the solution space of (1) is not additive. Hence, the above criteria are useless for verifying that the zero solution of (1) is attractive and stable, respectively. Will the attractivity guarantee the stability of the zero solution of (1)? In this talk, we give the answer to this question.

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