Effects of noise on a periodic solution of a system of nonlinear delay-differential equations in application to semiconductor lasers

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Semiconductor passively mode-locked (ML) lasers are used for generation of short periodic optical pulses with high repetition rate for application in telecommunication networks. In the last few years a nonlinear delay-differential equation (DDE) model [1] of a passively ML semiconductor laser gained popularity due to its ability to describe multi-mode regimes such as mode locking:

$$\gamma^{-1}\partial_t A(t) + A(t) = \sqrt{\kappa}e^{(1-i\alpha_g)G(t-T)/2-(1-i\alpha_q)Q(t-T)/2}A(t-T) + f_1(t), \tag{1}$$

$$\partial_t G(t) = g_0 - \gamma_g G(t) - e^{-Q}(e^{G(t)} - 1)|A(t)|^2 + f_2(t), \tag{2}$$

$$\partial_t Q(t) = q_0 - \gamma_q Q(t) - s(1 - e^{-Q(t)})|A(t)|^2 + f_3(t). \tag{3}$$

Here $\mathbf{f}(t) = \{f_1(t), f_2(t), f_3(t)\}^T$ is a non-autonomous perturbation term, which can be periodic [2], [3] or stochastic. In this work, we focus on the perturbation $\mathbf{f}(t) = \{\xi\eta(t), 0, 0\}^T$, where η is the white noise and $\xi \ll 1$. We are interested in the noise induced timing jitter, which is defined as the variance of the time interval between the pulses. We linearise the autonomous system near its ML periodic solution (A_0, G_0, Q_0) and project the perturbation term on the neutral eigenfunctions that correspond to the time shift and phase invariance of the autonomous equations. In order to compute the projection, we find numerically the solution (A_0, G_0, Q_0) and the adjoint neutral eigenfunctions with the help of DDE-BIFTOOL software [4]. We use asymptotic formulas to calculate the dependence of the timing jitter on parameters of the system with a special attention to parameter regions near instabilities of the ML regime.

References

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