## Steady compressible Navier-Stokes-Fourier system


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We consider the system of partial differential equations describing the steady flow of a compressible heat conducting fluid in a bounded three-dimensional domain

$$\begin{aligned} \operatorname{div}(\varrho \mathbf{u}) &= 0, \\ \operatorname{div}(\varrho \mathbf{u} \otimes \mathbf{u}) - \operatorname{div} \mathbf{S} + \nabla p &= \varrho \mathbf{f}, \\ \operatorname{div}(\varrho E \mathbf{u}) &= \varrho \mathbf{f} \cdot \mathbf{u} - \operatorname{div}(p\mathbf{u}) + \operatorname{div}(\mathbf{S}\mathbf{u}) - \operatorname{div} \mathbf{q} \end{aligned} \tag{1}$$

with $\varrho$ the density, $\mathbf{u}$ the velocity field, $\mathbf{S}$ the stress tensor (here we assume the fluid to be Newtonian with temperature dependent viscosity), $p$ the pressure, $\mathbf{f}$ the given volume force, $\mathbf{q}$ the heat flux and the total energy $E = \frac{1}{2}|\mathbf{u}|^2 + e$ with $e$ the internal energy. We assume the pressure law of the form $p(\varrho, \vartheta) \sim \varrho^\gamma + \varrho\vartheta$ with $\gamma > 1$ and the viscosities $\mu(\vartheta), \xi(\vartheta) \sim (1+\vartheta)^\alpha$, $\alpha \in [0,1]$.

We show the existence of a weak or variational entropy solution for the above model with internal energy fulfilling the Gibbs relation and the heat flux fulfilling the Fourier law $\mathbf{q} \sim (1+\vartheta)^m \nabla\vartheta$ with $\vartheta$ the temperature, $m = m(\gamma, \alpha) > 0$. It is an extension of the result in [1], where the special case $\alpha = 1$ was considered and it also extends the results from [2], where the method to treat $\alpha \in [0,1]$ was restricted to values $\gamma > \frac{3}{2}$. Note that the variational entropy solution exists for any $\gamma > 1$ while for the weak solution we need at least $\gamma > \frac{5}{4}$.

The solution to (1) is constructed for arbitrarily large sufficiently regular data. It is a joint work with Ondřej Kreml.

*References*


[1] *D. Jesslé, A. Novotný, M. Pokorný*: Steady Navier-Stokes-Fourier system with slip boundary conditions. Accepted to Math. Models Metods. Appl. Sci.

[2] *O. Kreml, Š. Nečasová, M. Pokorný*: On the steady equations for compressible radiative gas. Z. Angew. Math. Phys. *64* (2013), 539–571.