Direct and inverse problems for semilinear higher order ultraparabolic equation

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Let $\Omega \subset \mathbb{R}^n$ and $D \subset \mathbb{R}^l$ be bounded domains with the boundaries $\partial\Omega \in C^m$ and $\partial D \in C^1$; $x \in \Omega$, $y \in D$, $t \in (0,T)$, where $T > 0$ is a fixed number.

Denote: $G = \Omega \times D$, $Q_T = G \times (0,T)$, $\Sigma_T = \partial\Omega \times D \times (0,T)$, $S_T = \Omega \times \partial D \times (0,T)$.

In the domain Q_T we consider the equation

$$u_t + \sum_{i=1}^{l} \lambda_i(x,y,t) u_{y_i} + \sum_{|\alpha|=|\gamma|\le m} (-1)^{|\gamma|} D^\gamma (a_{\alpha\gamma}(x,y,t) D^\alpha u) + c(x,y,t)u + g(x,y,t,u) = \sum_{i=1}^{s} f_i(x,y,t) q_i(t) + f_0(x,y,t) \tag{1}$$

with the initial condition

$$u(x,y,0) = u_0(x,y), \quad (x,y) \in G, \tag{2}$$

and the boundary conditions

$$D^\alpha u|_{\Sigma_T} = 0,\ |\alpha| \le m-1, \qquad u|_{S_T^1} = 0. \tag{3}$$

Here $D^\alpha = \frac{\partial^{|\alpha|}}{\partial x_1^{\alpha_1} \ldots \partial x_n^{\alpha_n}}$, $|\alpha| = \alpha_1 + \cdots + \alpha_n$, $S_T^1 = \left\{(x,y,t) \in S_T : \sum_{i=1}^{l} \lambda_i(x,y,t) \cos(\nu, y_i) < \right\}$, ν is an outward unit normal vector to the surface S_T, function $g(x,y,t,u)$ satisfies a Lipschitz condition on the variable u.

Such results were obtained:

1) the solvability of the mixed problem (1)–(3) in Sobolev spaces;

2) the existence and the uniqueness of weak solution $(u(x,y,t), q_1(t), \ldots, q_s(t))$ for the inverse problem (1)–(3) with the overdetermination conditions

$$\int_{G_t} K_i(x,y) u(x,y,t)\,\mathrm{d}x\,\mathrm{d}y = E_i(t), \qquad t \in [0,T], \qquad i = 1, \ldots, s,$$

when $q_i(t)$ $(i = 1, \ldots, s)$ are unknown functions.