Hopf bifurcation for dissipative hyperbolic PDEs

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We consider boundary value problems for semilinear first order hyperbolic systems of the type

$$\partial_t u_j + a_j(x,\lambda)\partial_x u_j + b_j(x,\lambda,u) = 0,\ x \in (0,1),\ j = 1,\dots,n,$$
$$u_j(0,t) = \sum_{k=m+1}^{n} r_{jk} u_k(0,t), \quad j = 1,\dots,m,$$
$$u_j(1,t) = \sum_{k=1}^{m} r_{jk} u_k(1,t), \quad j = m+1,\dots,n$$

with smooth coefficient functions a_j and b_j such that $b_j(x,\lambda,0) = 0$. We state conditions for Hopf bifurcation, i.e. for existence, local uniqueness (up to phase shifts), smoothness and smooth dependence on λ of time-periodic solutions bifurcating from the zero stationary solution. Furthermore, we derive a formula which determines the bifurcation direction.

The proof is done by means of a Lyapunov-Schmidt reduction procedure. For this purpose, Fredholm properties of the linearized system and implicit function theorem techniques are used.

There are several distinguishing features of the proofs of Hopf bifurcation theorems for hyperbolic PDEs in comparison with those for parabolic PDEs or for ODEs: First, the question of Fredholm solvability of the linearized problem (in appropriate spaces of time-periodic functions) is essentially more difficult. Second, the question if a non-degenerate time-periodic solution of the nonlinear problem depends smoothly on the system parameters is much more delicate. And third, a sufficient amount of dissipativity is needed in order to prevent small denominators from coming up, and we present an explicit sufficient condition for that in terms of the data of the PDEs and of the boundary conditions.

This is a joint work with Irina Kmit.

References

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