Extreme solutions to a system of n nonlinear differential equations and regularly varying functions

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We consider the nonlinear differential system

$$x_i' = \delta a_i(t) F_i(x_{i+1}), \quad i = 1, \ldots, n, \quad t \in [a, \infty), \tag{S}$$

where x_{n+1} means x_1, $\delta \in \{-1, 1\}$, $n \geq 2$, a_i are continuous regularly varying (at infinity) functions, and F_i are continuous functions with $|F_i(|\cdot|)|$ being regularly varying (at infinity or at zero) with a positive index. The subhomogeneity condition is assumed. System (S) includes several extensively studied objects: For instance, the n-th order two-term nonlinear equation $x^{(n)} = p(t)|x|^\beta \operatorname{sgn} x$, or some of its generalized variants, or the second order system which arises out when studying positive radial solutions to the partial differential system $\operatorname{div}(\|\nabla u_i\|^{\lambda_i - 1} \nabla u_i) = \varphi_i(\|z\|) G(u_{i+1})$, $i = 1, \ldots, n$, $u_{n+1} = u_1$, or, after a certain modification, equations with a generalized Laplacian, e.g. of the form $(r(t)G(x'))' = p(t)F(x)$.

We study asymptotic behavior of extreme solutions to system (S). By the extreme solutions we mean the so called strongly decreasing resp. strongly increasing solutions, that is, eventually positive solutions whose all components tend to zero (for (S) with $\delta = -1$) resp. to infinity (for (S) with $\delta = 1$) as $t \to \infty$. We show that—under quite natural conditions—such solutions exist and are (all) regularly varying with known index. Moreover, we establish precise asymptotic formulas.

The results improve and extend some known results in various aspects. They are new even in certain well studied special settings.

This is a joint work with Serena Matucci (University of Florence).