Oscillation of a class of nonlinear neutral differential equations

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Neutral differential equations are used for modeling many problems arising in astrophysics, atomic physics, gas and fluid mechanics, etc. Therefore, analysis of the asymptotic behavior of solutions to such equations is important for applications. In particular, oscillatory and non-oscillatory behavior of solutions to various classes of nonlinear differential and functional differential equations has always attracted attention of researchers. In this paper, we are concerned with the oscillation of a class of nonlinear second-order neutral differential equations

$$(1) \qquad (r(t)((x(t) + p(t)x(t - \tau))')^\gamma)' + q(t)f(x(t), x(\sigma(t))) = 0,$$

where $t \geq t_0 > 0$, $\tau \geq 0$, and $\gamma \geq 1$ is a quotient of two odd positive integers. We suppose that

(H_1) $r \in C^1([t_0, +\infty), (0, +\infty))$;

(H_2) p, $q \in C([t_0, +\infty), [0, +\infty))$ and $q(t)$ is not identically zero for large t;

(H_3) $f \in C(\mathbb{R}^2, \mathbb{R})$ and $f(x, y)/y^\gamma \geq \kappa$, for all $y \neq 0$ and for some $\kappa > 0$;

(H_4) $\sigma \in C^1([t_0, +\infty), \mathbb{R})$, $\sigma(t) \leq t$, $\sigma'(t) > 0$, and $\lim_{t \to \infty} \sigma(t) = +\infty$.

Recently, Xu and Meng [1] established sufficient conditions for the oscillation of (1) assuming that

$$(2) \qquad p'(t) \geq 0 \qquad \text{and} \qquad \lim_{t \to +\infty} p(t) = A.$$

Further results in that direction were obtained by Ye and Xu [2] under the assumptions that

$$(3) \qquad p'(t) \geq 0 \qquad \text{and} \qquad \sigma(t) \leq t - \tau.$$

Our principal goal in this paper is to derive new oscillation criteria for equation (1) without requiring restrictive conditions (2) and (3). This is a joint work with Tongxing Li and Chenghui Zhang.

References

[1] *R. Xu, F. W. Meng*: Some new oscillation criteria for second order quasi-linear neutral delay differential equations. Appl. Math. Comput. *182* (2006), 797–803.

[2] *L. Ye, Z. Xu*: Oscillation criteria for second-order quasilinear neutral delay differential equations. Appl. Math. Comput. *207* (2009), 388–396.