

# Solvability of a one dimensional free boundary problem for adsorption phenomena

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This is a joint work with Prof. Toyohiko Aiki, Prof. Yusuke Murase and Prof. Ken Shirakawa.

We study the following system which is a mathematical model for adsorption phenomena in a small pore: Find a curve  $s$  on  $[0, T]$  ( $0 < T < \infty$ ) and a function  $u$  on  $Q_s(T)$  satisfying that

$$\begin{aligned} (1) \quad & 0 < s(t) < L \text{ for } 0 < t < T, \\ (2) \quad & \rho_v u_t - \kappa u_{xx} = 0 \text{ in } Q_s(T), \\ (3) \quad & u(t, L) = g(t) \text{ for } 0 < t < T, \\ (4) \quad & s'(t) = \alpha(s(t), u(t, s(t))) \text{ for } 0 < t < T, \\ (5) \quad & \kappa u_x(t, s(t)) = (\rho_w - \rho_v u(t, s(t)))s'(t) \text{ for } 0 < t < T, \\ (6) \quad & s(0) = s_0, \\ (7) \quad & u(x, 0) = u_0(x) \text{ for } s_0 < x < L, \end{aligned}$$

where  $Q_s(T) := \{(t, x); 0 < t < T, s(t) < x < L\}$ ,  $u$  is the humidity in the pore with depth  $L$ ,  $s = s(t)$  is the free boundary separating the water-drop zone and the vapor zone,  $\rho_w$  is the density of the water and  $\rho_v$  is the density of the vapor. Furthermore,  $\kappa$  is a positive constant,  $\alpha$  is a Lipschitz continuous function on  $R^2$ ,  $g$  is a given function and  $s_0$  and  $u_0$  are initial data. We denote by  $P := P(s_0, u_0, g)$  the above system (1) ~ (7). Our modeling process of  $P$  is mentioned in [1].

Main results of the present talk are following:

**Theorem 1** (Time-local existence). *Let  $T > 0$ . Then under suitable assumptions problem  $P$  has a unique strong solution on  $[0, T']$  for some  $T' \in (0, T]$ .*

**Theorem 2** (Time-global existence). *Either the following (i) or (ii) holds for problem  $P$  under suitable assumptions:*

- (i) *For any  $T > 0$ , problem  $P$  has a unique solution  $\{s, u\}$  on  $[0, T]$ .*
- (ii) *There exists some  $T^* > 0$  such that  $s(t) \rightarrow L$  as  $t \rightarrow T^*$ .*

## References

- [1] *T. Aiki, Y. Murase, N. Sato, K. Shirakawa: A mathematical model for a hysteresis appearing in adsorption phenomena. To appear in RIMS Kôkyûroku in published in 2013.*