BMO estimates for p-parabolic systems

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We present results of the non-linear Calderon Zygmund theory for the parabolic p-Laplace. We consider local solutions of the system

$$\partial_t u - \operatorname{div}(|\nabla u|^{p-2}\nabla u) = -\operatorname{div}(|F|^{p-2}F). \tag{1}$$

It is the concern of the non-linear Calderon Zygmund theory to transfer qualities from the right hand side F to ∇u.

Let us first consider the linear stationary situation, i.e. Poisson's equation. In this case $F \mapsto \nabla u$ is a singular integral operator such that a lot of regularity transfers from F to ∇u. If $p \neq 2$ things are similar in the stationary case. Iwaniec proved in [3] that

$$M_p^\sharp(\nabla u)(x) \leq c M_p(F) + \delta M_p(\nabla u),$$

for almost every x. Here M_q is the Hardy Littlewood and $M_q^\sharp$ is the Fefferman Stein Maximal operator with power q. From this inequality he gains by standard harmonic analysis that the mapping $F \mapsto \nabla u$ is bounded from $L^{pq}(\mathbb{R}^n) \to L^{pq}(\mathbb{R}^n)$, for all $1 < q < \infty$. In [2] we proved the borderline space ($q \to \infty$), which is the space of bounded mean oscillation (BMO). We showed that

$$M_1^\sharp(|\nabla u|^{p-2}\nabla u)(x) \leq c\||F|^{p-2}F\|_{\mathrm{BMO}(\mathbb{R}^n)}.$$

In the parabolic setting things are different. If u is a solution to (1), then λu is not necessarily a solution. However, some regularity can be shown. The transference of integrability properties was shown by Acerbi and Mingione [1]. Recently the parabolic version of the BMO case was proved in [4], i.e. if $p \geq 2$ and $F \in L^\infty(\mathrm{BMO})$, then $u \in L^\infty(\mathcal{C}^1)$, where $\mathcal{C}^1$ is the 1-Hölder Zygmund space.

References

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