Diffusion-type dynamic equations with discrete-space domains

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We investigate the properties of the dynamic equation

$$u^{\Delta_t}(x,t) = au(x+1,t) + bu(x,t) + cu(x-1,t), \quad x \in \mathbb{Z},\ t \in \mathbb{T},$$

where $\mathbb{T}$ is an arbitrary time scale and u^{Δ_t} denotes the Δ-derivative of u with respect to t.

When $\mathbb{T} = \mathbb{R}$, this equation generalizes the space-discretized version of the classical diffusion equation. For $\mathbb{T} = \mathbb{Z}$, the equation describes the one-dimensional (not necessarily symmetric) random walk.

We discuss the existence and (non)uniqueness of solutions to initial-value problems, superposition principle, space sum preservation, and maximum and minimum principles. It turns out that the graininess of $\mathbb{T}$ influences the behavior of solutions in a substantial way.

This is a joint work with Petr Stehlík (University of West Bohemia, Pilsen).