Symmetry breaking of solutions of non-cooperative elliptic systems

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The aim of my talk is to consider a problem of symmetry breaking of solutions of non-cooperative elliptic systems of the form:

$$\begin{cases} -\Delta w_1 = \nabla_{w_1} F(w_1, w_2) + f_1 & \text{in } \Omega \\ \Delta w_2 = \nabla_{w_2} F(w_1, w_2) + f_2 & \text{in } \Omega \\ \frac{\partial w_1}{\partial \nu} = \frac{\partial w_2}{\partial \nu} = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $\mathbb{R}^n$ is an orthogonal representation of a compact Lie group G, $\Omega \subset \mathbb{R}^n$ is an open, bounded, G-invariant set with a smooth boundary and $F \in C^2(\mathbb{R}^2, \mathbb{R})$. That is we discuss the existence of a G-symmetric function (f_1, f_2) such that there is a K-symmetric solution (w_1, w_2) of system (1), where K is a closed subgroup of G. If such a solution exists, we say that occurs a symmetry breaking of solutions of problem (1).

Solutions of such systems can be considered as critical points of strongly indefinite functionals. As the main topological tool we use the degree for G-invariant strongly indefinite functionals, see [2]. We obtain simultaneously a symmetry breaking and a global bifurcation phenomenon. This is a generalization of the result due to E. N. Dancer, see [1].

As an application of the method we consider a case of the special orthogonal group and its non-trivial subgroup. We obtain a connected set of radial functions for which exist non-radial solutions of the problem (1).

References

[1] *E. N. Dancer*: Breaking of symmetries for forced equations. Math. Ann. *262* (1983), 473–486.

[2] *A. Gołębiewska, S. Rybicki*: Global bifurcations of critical orbits of G-invariant strongly indefinite functionals. Nonlinear Anal. *74* (2011), 1823–1834.

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