Effective flow of quasi-newtonian fluid through a domain with a slightly rough bottom

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We consider the quasi-Newtonian flow in a domain with a periodic rough bottom Γ_ε of period ε and amplitude δ_ε such that $\delta_\varepsilon \ll \varepsilon$ (see [1] for the case $\delta_\varepsilon = \varepsilon$). The flow is described by the 3D incompressible Navier-Stokes system where the viscosity is given by the non linear power law which is widely used for dilatant fluids (shear thickening). The viscosity η_r as a function of the shear rate is given by

$$\eta_r(D(u_\varepsilon)) = \tau |D(u_\varepsilon)|^{r-2}, \quad r > 2,$$

where u_ε is the velocity of the fluid, $\tau > 0$ and $2 < r < \infty$ (see [2] for the case $r = 2$). The power law consistency index τ and the power law exponent r are material parameters.

We assume that the fluid satisfies on Γ_ε the slip condition given by Navier's law

$$u_\varepsilon \cdot \nu = 0, \quad (|D(u_\varepsilon)|^{r-2} D(u_\varepsilon) \cdot \nu + \gamma u_\varepsilon)_\tau = 0 \text{ on } \Gamma_\varepsilon,$$

with $\gamma \geq 0$ the friction coefficient. Letting $\varepsilon \to 0$, we obtain three different macroscopic models depending on the magnitude of δ_ε with respect to $\varepsilon^{\frac{2r-1}{r}}$. In each case we identify the roughness-induced effects and prove corrector results for the velocity and the pressure.

References

[1] *D. Bucur, E. Feireisl, Š. Nečasová*: Influence of wall roughness on the slip behavior of viscous fluids. Proceedings of the Royal Society of Edinburgh: Section A Mathematics *138* (2008), 957–973.

[2] *J. Casado-Díaz, M. Luna-Laynez, F. J. Suárez-Grau*: Asymptotic behavior of a viscous fluid with slip boundary conditions on a slightly rough wall. Math. Models Methods Appl. Sci. *20* (2010), 121–156.