# Discrete model of the Dirac-Kähler equation

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In this work we present a new geometric discretization of the Dirac-Kähler equation. Let $\Omega$ be a inhomogeneous differential form in Minkowski space with metric signature $(+,-,-,-)$. Such form is a sum of all differential $r$-forms and we have $\Omega=\sum_{r=0}^{4}\overset{r}{\omega}$. The Dirac-Kähler equation on exterior forms is given by

$$(d-\delta)\Omega=m\Omega, \tag{1}$$

where $d$ is the exterior differential, $\delta=-*d*$ is the formal adjoint of $d$ (the codifferential) and $*$ is the Hodge star operator. Here $m$ is a real nonnegative constant (the particle mass). Equation (1) is the generalization of the Dirac equation. It is easy to show that Equation (1) is equivalent to the following equations

$$\begin{aligned}
-\delta\overset{1}{\omega}&=m\overset{0}{\omega},\\
d\overset{0}{\omega}-\delta\overset{2}{\omega}&=m\overset{1}{\omega},\\
d\overset{1}{\omega}-\delta\overset{3}{\omega}&=m\overset{2}{\omega},\\
d\overset{2}{\omega}-\delta\overset{4}{\omega}&=m\overset{3}{\omega},\\
d\overset{3}{\omega}&=m\overset{4}{\omega}.
\end{aligned}$$

We propose a discretization scheme based on the use of the differential form language in which the exterior derivative $d$, the Hodge star operator $*$ and the exterior product $\wedge$ of differential forms are replaced by their discrete analogies. The algebraic relations between these operators are captured in the proposed discrete model. This approach was initiated by Dezin in [1]. Using the combinatorial model of Minkowski space described in [2] we construct an intrinsically defined discrete analog of Equation (1). We show that geometric properties of the Dirac-Kähler system that hold in the smooth setting also hold in the discrete case. Discrete analogs of Dirac type operators are also studied.

*References*

[1] *A. A. Dezin*: Multidimensional analysis and discrete models. CRC Press, Boca Raton, 1995.

[2] *V. Sushch*: Discrete model of Yang-Mills equations in Minkowski space. Cubo Math. J. *6* (2004), 35–50.