Critical case of nonlinear Schrödinger equations with inverse-square potentials

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In this talk we consider the following Cauchy problem for nonlinear Schrödinger equations with inverse-square potential:

$$(\mathbf{NLS})_a \qquad \begin{cases} i\dfrac{\partial u}{\partial t} = -\Delta u + \dfrac{a}{|x|^2}u + f(u) & \text{in } \mathbb{R}\times\mathbb{R}^N, \\ u(0,x) = u_0(x) & \text{on } \mathbb{R}^N, \end{cases}$$

where $i = \sqrt{-1}$, $N \geq 3$ and $a \geq -(N-2)^2/4$. The feature for $(\mathbf{NLS})_a$ is the presence of a strongly singular potential $a|x|^{-2}$; note that $-\Delta$ and $a|x|^{-2}$ are the same scale symmetry and hence scaling argument can not be applied to $P_a := -\Delta + a|x|^{-2}$. The restriction on a follows from the selfadjointness of P_a in the sense of form-sum in $L^2(\mathbb{R}^N)$.

Applying energy methods for abstract nonlinear Schrödinger equations (see Okazawa-S.-Yokota [1]) we can prove the weak solvability of $(\mathbf{NLS})_a$. For exapmle, let $f(u) := \lambda|u|^{p-1}u$. Assume that either $1 \leq p < (N+2)/(N-2)$ $(\lambda > 0)$ or $1 \leq p < 1 + 4/N$ $(\lambda < 0)$. Then we can solve $(\mathbf{NLS})_a$ for every $u_0 \in H^1(\mathbb{R}^N)$ $(a > -(N-2)^2/4)$.

References

[1] *N. Okazawa*, *T. Suzuki*, *T. Yokota*: Energy methods for abstract nonlinear Schrödinger equations. Evol. Equ. Control Theory *1* (2012), 337–354.