Oscillation criteria for some third order ordinary differential equations

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In this work we investigate some oscillation criteria for problems of the type

$$(1) \qquad \begin{cases} \text{(i)} \quad u'''(t) + c(t)u'(t) + h(t,u) = 0, \ t > t_0 \geq 0; \ u(t_0) = u''(t_0) = 0; \\ \text{(ii)} \quad \text{where } c \in C^1(\mathbb{R},(0,\infty)), \ h \in C(\mathbb{R}\times\mathbb{R},\mathbb{R}) \\ \text{(iii)} \quad \text{with } h(\cdot,0) = 0 \text{ and } \forall\, s \in \mathbb{R}\setminus\{0\}, \ sh(t,s) > 0. \end{cases}$$

Here $\{\cdot\}'$ stands for $\dfrac{\mathrm{d}\{\cdot\}}{\mathrm{d}t}$. We recall that a function u is said to be oscillatory if it has a zero in any exterior domain $\Omega_T := (T,\infty)$; $T \geq 0$. The equation (1)(i) is odd in the sense that whenever w is a solution, so is $-w$. Therefore a solution u will be said to be non oscillatory if it is eventually strictly positive (i.e. there is an $S > 0$ such that $u > 0$ in Ω_S). In the sequel a nodal set of a function v (or v^+) would mean any interval $D(v)(D(v^+)) = (t,s)$ such that $v(t) = v(s) = 0$ with $v \neq 0$ $(v > 0)$ in (t,s). A function u will be said to be a solution of (1) if it is in $C^3(\mathbb{R})$ and satisfies the problem. Here an equation will be said to be oscillatory if its bounded solutions are oscillatory. It is obvious that if ϕ is oscillatory so will be ϕ'. Finally if $h(\cdot,\cdot) \equiv 0$ in the equation (1)(i), u''' and u' are simultaneously oscillatory or non-oscillatory. In the sequel we assume that

$$(\mathbf{H}) \qquad \begin{cases} \text{(h1)} \quad h(t,S) = q(t)f(S) \quad \text{where} \quad f \in C(\mathbb{R}), \ Sf(S) > 0 \quad \forall\, S \neq 0 \quad \text{and} \quad f(0) = 0; \\ \text{(h2)} \quad c, q \in C(\Omega_{t_0},(0,\infty)) \quad \text{for some} \quad t_0 \geq 0. \end{cases}$$

We use some integral properties and some comparison methods, using mainly some Picone-type identities for our results. This lays down a way of tackling some higher order equations using some lower order associate equations.