

Convergence for a 2D elliptic problem with large exponent in nonlinearity

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In this talk, we are concerned with the problem

$$(E_p) \quad -\Delta u = K(x)u^p \quad \text{in } \Omega, \quad u > 0 \quad \text{in } \Omega, \quad u = 0 \quad \text{on } \partial\Omega,$$

where Ω is a smooth bounded domain in \mathbb{R}^2 , $K \in C^1(\overline{\Omega})$, $\inf_{\overline{\Omega}} K > 0$ is a given positive weight function, and $p > 1$ is a nonlinear exponent.

Let $\{u_p\}$ be a sequence of solutions to (E_p) , not necessarily least energy ones. The main purpose of this talk is to investigate the asymptotic behavior of general solutions u_p when the nonlinear exponent p gets large.

Let $\{p_n\}$ be a sequence of exponents with $p_n > 1$, $p_n \rightarrow +\infty$, and $\{u_{p_n}\}$ be a solution sequence of (E_p) for $p = p_n$. In this talk, we prove that along a subsequence $\{p_n\}$, mass quantization occurs in the sense that

$$p_n \int_{\Omega} K(x)u_{p_n}^{p_n} dx \rightarrow 8\pi\sqrt{e}N, \quad N \in \mathbb{N} \cup \{+\infty\}.$$

Furthermore, we have the entire blow-up if $N = +\infty$, or, N -points concentration holds if $N \in \mathbb{N}$, in the sense that there exists a set of N points $\mathcal{S} = \{x_1, \dots, x_N\} \subset \Omega$ such that, as $p_n \rightarrow \infty$,

$$\begin{aligned} \lim_{n \rightarrow \infty} \sup_{B_r(x_i)} u_{p_n} &= \lim_{n \rightarrow \infty} \|u_{p_n}\|_{L^\infty(\Omega)} = \sqrt{e} \quad (r > 0 \text{ small}), \\ p_n K(x)u_{p_n}^{p_n} &\overset{*}{\rightharpoonup} 8\pi\sqrt{e} \sum_{i=1}^N \delta_{x_i} \text{ in the sense of measures on } \overline{\Omega}, \\ p_n u_{p_n} &\rightarrow 8\pi\sqrt{e} \sum_{i=1}^N G(\cdot, x_i) \text{ in } C_{\text{loc}}^1(\overline{\Omega} \setminus \mathcal{S}). \end{aligned}$$

Also we obtain a characterization of each concentration point as a critical point of some function defined by the Green function and the coefficient function K .

These results are obtained by using ideas and techniques from the recent paper by S. Santra and J. C. Wei ([1]) with suitable modifications.

References

- [1] *S. Santra, J. C. Wei: Asymptotic behavior of solutions of a biharmonic Dirichlet problem with large exponents. J. Anal. Math. 115 (2011) 1–31.*