Morse index and symmetry-breaking for positive solutions of one-dimensional Hénon type equations

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The following two-point boundary value problem

$$\begin{cases} u'' + (|x|^l + \lambda)u^p = 0, & x \in (-1,1), \\ u(-1) = u(1) = 0 \end{cases}$$

is considered, where $l \geq 0$, $\lambda \geq 0$ and $p > 1$. There always exists a positive even solution. This problem has also the positive least energy solution. It is known that the Morse index of the least energy solution equals 1. Therefore, if we prove that the Morse index of positive even solutions is greater than or equal to 2, then the positive least energy solution is non-even, and hence symmetry-breaking phenomena occur. On the other hand, it is shown that if the Morse index of every positive solution equals 1, then the positive solution is unique and even. In this talk, the Morse indices of positive solutions are studied. By applying it, relations between (l, λ, p) and the number of positive solutions will be given.