Energy-dissipations in multidimensional Kobayashi-Warren-Carter models of grain boundaries

Hiroshi Watanabe
Department of General Education, Salesian Polytechnic, Tokyo, Japan
h-watanabe@salesio-sp.ac.jp

This study is based on a joint work with Ken Shirakawa, Chiba Univ., Japan, and Salvador Moll, Univ. Valencia, Spain.

Let $\Omega \subset \mathbb{R}^N$ be a bounded domain with a smooth boundary $\partial\Omega$, and let $\nu_{\partial\Omega}$ be the unit outer normal on $\partial\Omega$. In this talk, a coupled system of two parabolic type initial-boundary value problems, denoted by (S), is considered. This system is formally described as follows:

(S):

$$(1)\qquad \begin{cases} \eta_t - \Delta\eta + g(\eta) + \alpha'(\eta)|D\theta| = 0 \ \text{ in } Q := (0,\infty)\times\Omega, \\ \nabla\eta \cdot \nu_{\partial\Omega} = 0 \ \text{ on } \Sigma := (0,\infty)\times\partial\Omega, \\ \eta(0,x) = \eta_0(x), \quad x \in \Omega, \end{cases}$$

$$(2)\qquad \begin{cases} \alpha_0(\eta)\theta_t - \mathrm{div}\Big(\alpha(\eta)\dfrac{D\theta}{|D\theta|}\Big) = 0 \ \text{ in } Q, \\ \alpha(\eta)\dfrac{D\theta}{|D\theta|}\cdot\nu_{\partial\Omega} = 0, \quad \text{on } \Sigma, \\ \theta(0,x) = \theta_0(x), \quad x \in \Omega, \end{cases}$$

where $g \in W^{1,\infty}(0,1)$ is a given perturbation, $\alpha_0 \in W^{1,\infty}(0,1)$ and $\alpha \in C^2[0,1]$ are given positive-valued mobilities, and α' is the differential of α.

Recently, the method of mathematical treatment was developed in [1], and in the reference, the existence of solutions was reported as the main theorem. In this regard, we set the objective of this talk to ensure that:

- ▷ the energy dissipation reproduced by (S),
- ▷ the large-time behavior, i.e. the association between solutions in large time and the steady-state problem for (S).

References

[1] *S. Moll, K. Shirakawa*: Existence of solutions to the Kobayashi-Warren-Carter system. In preparation.