Boundedness of solutions to parabolic-elliptic Keller-Segel systems

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This talk is concerned with the boundedness in the following parabolic-elliptic Keller-Segel system:

$$\text{(KS)} \qquad \begin{cases} \dfrac{\partial u}{\partial t} = \Delta u - \nabla \cdot (u\chi(v)\nabla v) & \text{in } \Omega \times (0,\infty), \\ \quad 0 = \Delta v - v + u & \text{in } \Omega \times (0,\infty), \\ \dfrac{\partial u}{\partial n} = \dfrac{\partial v}{\partial n} = 0 & \text{in } \partial\Omega \times (0,\infty), \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N$ ($N \in \mathbb{N}$) with smooth boundary $\partial\Omega$. Assume that the initial data u_0 satisfies

$$u_0 \in C(\overline{\Omega}),\ u_0 \geq 0,\ \int_\Omega u_0 > 0.$$

The global existence and L^∞-boundedness of classical solutions to (KS) are established when (KS) has a generalized chemotactic sensitivity function $\chi(v)$. The result improves [1, Theorem 2] in which the global existence without L^∞-boundedness is shown in the special case $\chi(v) = \frac{\chi_0}{v}$. The key lies in lower bounds for v as in [2, Lemma 3.1].

This is a joint work with Kentarou Fujie and Michael Winkler.

References

[1] *P. Biler*: Global solutions to some parabolic-elliptic systems of chemotaxis. Adv. Math. Sci. Appl. *9* (1999), 347–359.

[2] *T. Hillen, K. Painter, M. Winkler*: Convergence of a cancer invasion model to a logistic chemotaxis model. Math. Models Methods Appl. Sci. *23* (2013), 165–198.