# Method of lines for parabolic stochastic functional partial differential equations

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We consider the following initial value problem for stochastic functional partial differential equation

$$\begin{cases} \frac{\partial}{\partial t}u(t,x) - \frac{\partial^2}{\partial x^2}u(t,x) = f(t,x,u_t) + g(t,x,u_t)\dot{B}_t \\ u(t,x) = \varphi(t,x) \quad \text{for } (t,x) \in D_0, \end{cases} \tag{1}$$

for $(t,x) \in [0,T] \times \mathbb{R}$, where

$$\varphi : [-r,0] \times \mathbb{R} \to \mathbb{R}^n$$
$$f : \mathrm{dom}(f) \to \mathbb{R}^n, \quad g : \mathrm{dom}(g) \to \mathbb{R}^{1\times p}$$
$$\mathrm{dom}(f) = \mathrm{dom}(g) := [0,T] \times \mathbb{R} \times \mathcal{C}(D_0, \mathbb{R}).$$

Problem (1) is discretized in size $s$ as follows. We introduce a mesh on $\mathbb{R}$ with the discretization step $x^{(i)} = i \cdot h$, $h > 0$, $i \in \mathbb{Z}$.

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}t}u^{(i)}(t) - \frac{u^{(i+1)}(t) - 2u^{(i)}(t) + u^{(i-1)}(t)}{h^2} \\ = f(t, h\cdot i, u_t) + g(t, h \cdot i, u_t)\dot{B}_t \quad \text{for } t \in [0,T], \quad i \in \mathbb{Z} \\ u^{(i)}(t) = \varphi^{(i)}(t) \quad \text{for } i \in \mathbb{Z}, \quad t \in [-r, 0]. \end{cases} \tag{2}$$

We prove stability criteria for the method of lines corresponding to parabolic stochastic functional differential equations driven by Brownian motion.

*References*

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